

## Introduction

Neutron stars are very dense objects formed when massive stars come to the end of their lives. Type I X-ray bursts are explosions which occur on the surfaces of some neutron stars [1, 2, 8]. It is believed the explosion begins in a spot in the liquid surface layer, before rapidly spreading across the entire surface, burning as it goes [3, 6]. By modelling this, we can infer neutron star properties such as radius and magnetic field, which are difficult to measure directly but are crucial for understanding the stars' interior physics [4, 5, 9]. My work involves investigating how burning spreads across the surface.

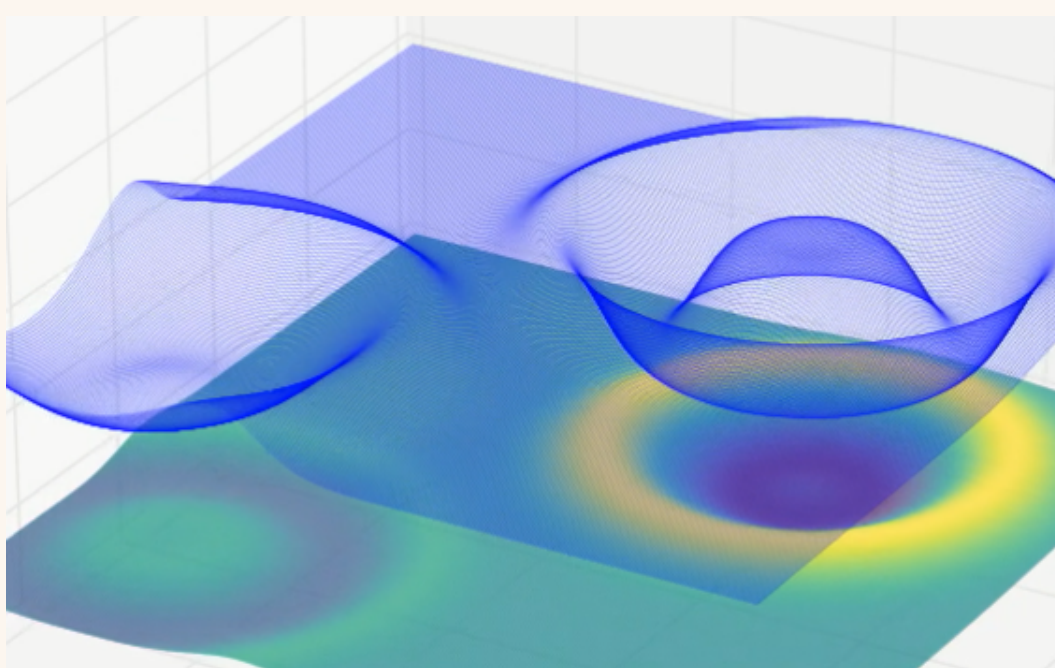
The neutron stars we're interested in are in orbit about low mass stars like our Sun. They pull matter from their companions (see figure 2), and this material makes its way onto the surface, forming a liquid ocean layer primarily consisting of hydrogen and helium. Eventually, the density and temperature in the ocean become high enough that nuclear fusion begins. This is a runaway process, and the nuclear burning very quickly (within a few seconds) spreads across the entire surface of the star. This is observed as a spike in the X-ray radiation of the star.

## Multiscale modelling

When modelling the system, it's important to account for the fact that the physics works across a wide range of scales. The large scale motion of the burning front is dominated by the Coriolis force (similar to hurricanes on Earth), whereas the small scale physics is dominated by the effects of turbulent burning [7].

We deal with these differences by taking approximations of the full fluid equations in certain limits. To model the large scale features of the system, I use the *shallow water equations*, which ignore variations in the vertical direction, modelling the ocean as a 2d surface. When looking at the small scale features, I instead use the *low Mach number* limit, which filters out fast moving sound waves. In order to capture the physics across the entire range of scales, I join these different models together to produce a *multiscale model* which solves these different sets of equations simultaneously.

## Shallow water equations



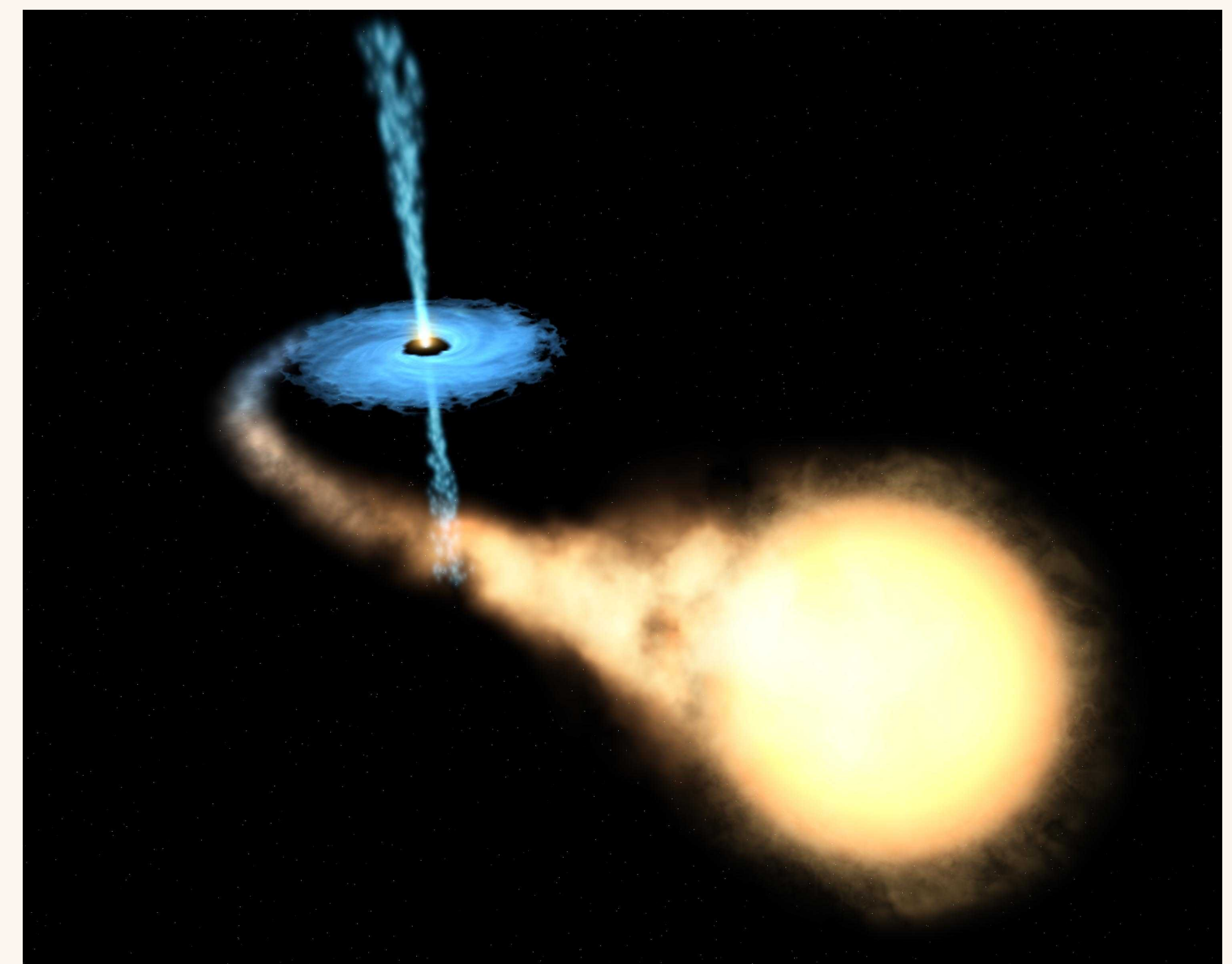
**Figure 1:** Simulation of ripples in a fluid using the shallow water equations.

This greatly simplifies the equations we need to solve, and significantly speeds up simulations. We can use this approximation to model large scale features as the ocean depth is much less than the star's radius.

The shallow water equations are often used to model weather systems on Earth. Here, I have extended them to include the strong, general relativistic gravitational field found on the surfaces of neutron stars, and to include reaction terms so we can model burning.

The *shallow water equations* are an approximation of the fluid equations where we ignore variations in the vertical direction, allowing us to model a 3d ocean as a 2d surface. This greatly simplifies the equations we need to solve, and significantly speeds up simulations. We can use this approximation to model large scale features as the ocean depth is much less than the star's radius.

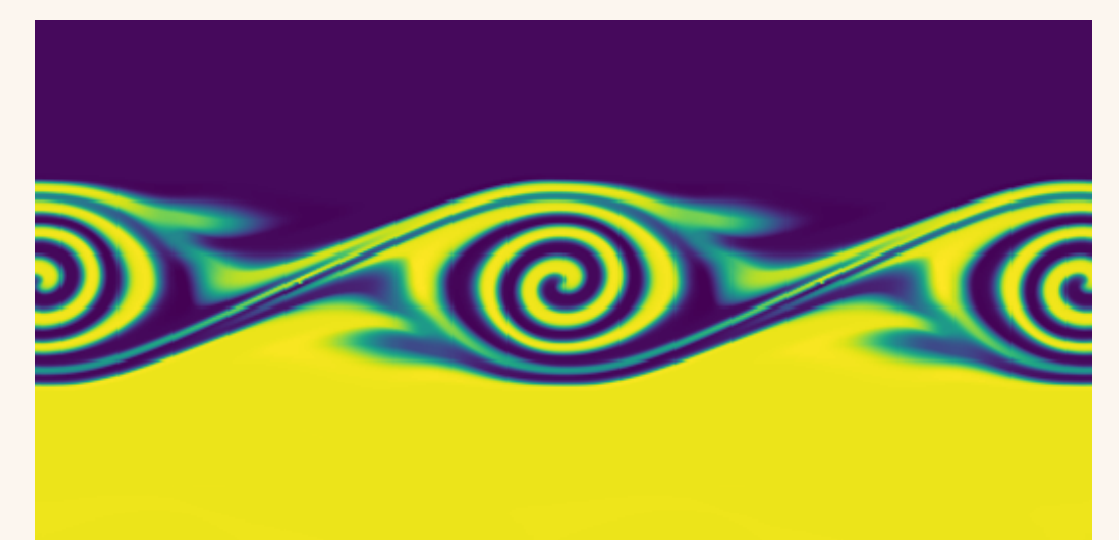
## Neutron star orbiting a low mass star



**Figure 2:** Neutron star in orbit about a low mass star [NASA]. This is known as a low mass X-ray binary (LMXB).

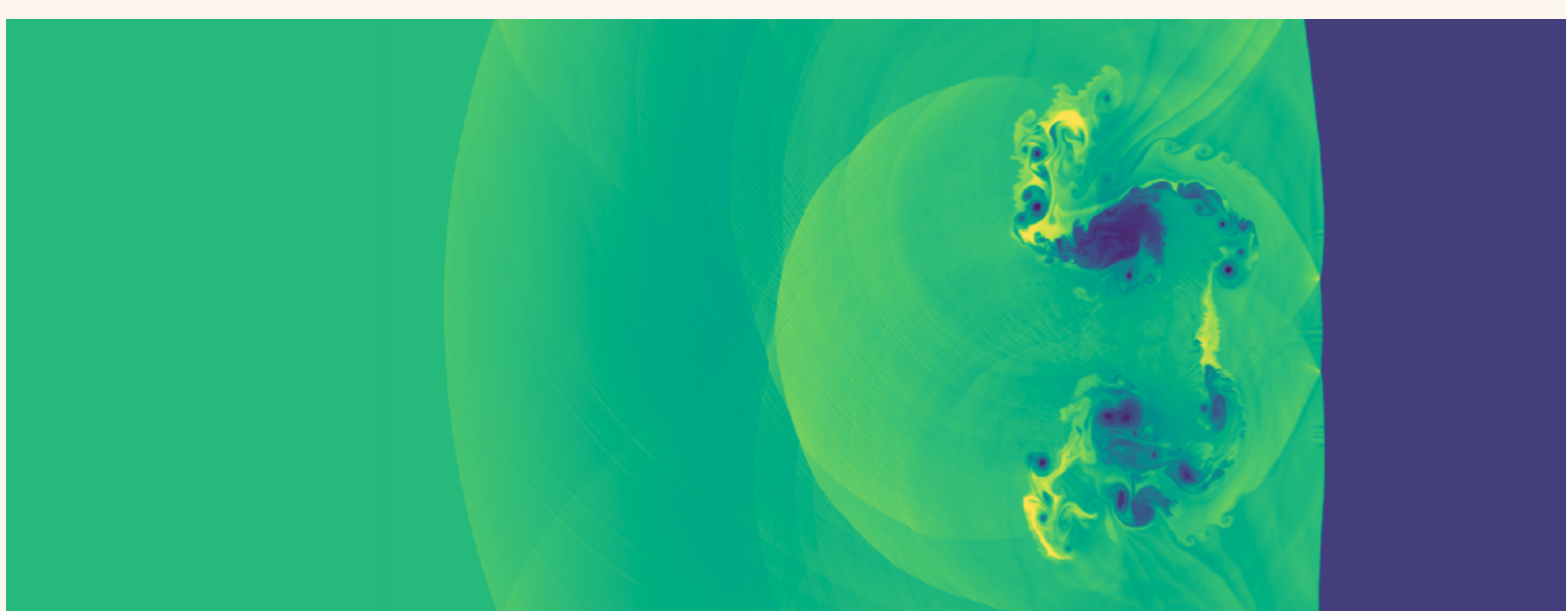
## Low Mach number approximation

The burning front moves very slowly compared to the sound waves in the ocean, which can severely restrict our simulations when we are modelling smaller features. We can get around this by using the *low Mach number approximation*, which effectively filters out the sound waves, allowing us to focus instead on the slower moving features we are interested in. This approximation has the benefit of significantly speeding up our calculations, allowing us to produce higher resolution simulations.



**Figure 3:** Simulation of the Kelvin-Helmholtz instability using the low Mach number approximation. Two layers of fluid move in opposite directions and swirling vortices form at their interface.

## Summary



**Figure 4:** Simulation of a relativistic shock wave hitting a rotating burning bubble.

I am modelling X-ray bursts on neutron stars, a type of explosion that occurs in the stars' oceans which we observe as a spike in their X-ray radiation. This will allow us to better calculate other properties, such as the radius and magnetic field. These are important when investigating the interior structure of neutron stars, which is currently poorly understood and where direct observations are impossible. To implement this, I have developed a *multiscale model* which joins together different physical models (including the shallow water equations and low Mach number approximation) to better capture the physics at different scales.

## References

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Background image: Crab nebula with neutron star at centre [NASA]